

**STUDY MATERIAL**  
**Building Mathematical Ability**

1. Natural numbers are counting numbers 1, 2, 3, 4, .....
2. Every natural number has one and only one successor.
3.  $n + 1$  is the successor of natural number  $n$
4. 10 is the successor of 9.
5. Every natural number except 1 is a successor of a number.
6. Addition and multiplication of two natural numbers is commutative.  
i.e.  $m + n = n + m$  and  $m \times n = n \times m$
7. Successor of a natural number is Unique.
8. If  $m$  and  $n$  are two natural numbers then any one of the following is true  
i.e.  $m = n$ , or  $m < n$  or  $m > n$
9. If  $m + n = l + n$  then  $m = l$   
If  $n + m = n + l$  then  $m = l$
10. If  $mn = ln$  then  $m = l$  provided  $n \neq 0$  also  $nm = nl$  then  $m = l$  provided  $n \neq 0$
11. Sum of first  $n$  natural numbers, i.e.  $1+2+3+ \dots + n = \frac{n(n+1)}{2}$
12. The number '0' is represented by ordered pairs (1,1) (2,2) (3,3) (4,4) .....  $(n, n)$
13. All positive natural numbers 1, 2, 3, 4 ....., all negative of natural numbers -1, -2, -3, -4 ..... and '0' are called integers.  
i.e.  $I = \{ \dots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \dots \}$  is the Set of integers.
14. Sum and product of two integers is an integer.
15. .... -8, -6, -4, -2, 0, 2, 4, 6, 8, 10 ..... are called even integers.
16. .... -7, -5, -3, -1, 1, 3, 5, 7, 9, ..... are called odd integers.
17. Sum and difference of two even integers is an even integer.
18. Sum and difference of two odd integers is an even integer.
19. Every natural number is an integer but every integer is not a natural number.
20.  $m < n$  if and only if  $m - n$  is negative.
21. If  $r$  is the remainder when  $m$  divided by  $n$  ( $m > n$ ) then  $r < n$ .
22. If  $n$  divides  $m$  then remainder is Zero.
23. Any number of the form  $P/q$  where  $p$  and  $q$  are integers and  $q \neq 0$  is called a rational number if the ratio terminates or, repeats.
24. Two rational numbers  $\frac{p}{q}, \frac{r}{s}$  are equal if  $ps = qr$
25. Every integer is a rational number with its denominator 1.
26. Sum and product of rational numbers  $\frac{p}{q}$  and  $\frac{r}{s}$  is a rational number  
i.e.  $\frac{p}{q} + \frac{r}{s} = \frac{ps+qr}{qs}$   
and  $\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}$
27. A nonterminating ratio  $\frac{p}{q}$ , where  $p, q$  are integers and  $q \neq 0$  is called an irrational number.
28. All rational and irrational numbers together called real numbers.
29. Every rational number is a real number but every real number is not a rational number.
30. Sum, difference, product, division of two real numbers is a real numbers.
31. A number of the form  $Z = a + ib$ , where  $a$  and  $b$  are real numbers and  $i = \sqrt{-1}$  is called a complex number.

32. Every real number is a complex number.
33. A number divisible by itself only is called a prime number.  
Ex. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 .....
34. Only one even prime is 2 and other prime numbers are odd.
35. Opening Code of an electronic device is Unique.
36. Prime numbers are used for encryption purpose.
37. A quantity  $x$  which can assume any value is called variable.
38. A first degree equation in variables  $x$  and  $y$  is called linear equation. i.e.  $ax + by + c = 0$  is a linear equation in  $x$  and  $y$  where  $a, b, c$  are constant.
39.  $ax + by + c \leq 0$  or  $ax + by + c \geq 0$  are called linear inequations in  $x$  and  $y$ .

40. Arrangement of numbers in rows and columns is called a matrix.

is  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is a  $2 \times 2$  matrix ( i.e. two rows and two columns)

Horizontal arrangements are called rows and vertical arrangements are called columns.

41.  $[a \ b]$  is a  $1 \times 2$  matrix or, row matrix of order  $1 \times 2$ .

$\begin{bmatrix} a \\ c \end{bmatrix}$  is a  $2 \times 1$  matrix or, column matrix of order  $2 \times 1$

42. Determinant of matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is given by  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

43. Matrix is called singular if its determinant is zero Ex. For the matrix  $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

its determinant =  $\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 2 \times 6 - 4 \times 3 = 12 - 12 = 0$

Thus, the matrix  $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$  is singular

44. matrix is called non singular if its determinant is non zero

Ex. For the matrix  $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$  its determinant =  $\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix} = 3 \times 6 - 4 \times 5 = 18 - 20 = -2 \neq 0$

The matrix  $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$  is non singular

45. Two linear equations

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

together, called simultaneous equations

And for these equations the matrix  $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$  is called coefficient matrix. The solution to these equations exists if the coefficient matrix is non singular.

46. Solution to simultaneous equations

$$3x + 2y = 5$$

$$5x + 4y = 6 \text{ exists, as}$$

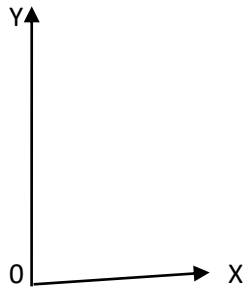
determinant of the coefficient matrix  $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix} = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 12 - 10 = 2 \neq 0$

47. Solution of the linear equation  $x + y = c$  is not unique as the equation is satisfied by more than one pair values of  $x$  and  $y$ .

48.  $x + y = 10$  has no Unique solution as  $x = 1, y = 9; x = 2, y = 8; x = 3, y = 7 \dots \dots x = 9, y = 1$  are all solutions of this equation.

49.  $x - y = 10$  or,  $y = x - 10$  has no unique solution.

50. Y

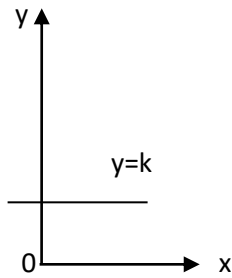


$xoy$  represents rectangular axes system. The horizontal line  $ox$  is called  $x$  – axis, vertical line  $oy$  is called  $y$  – axis and point  $O$  is called origin.

Equation of  $x$  axis i.e.  $ox$  line is  $y = 0$

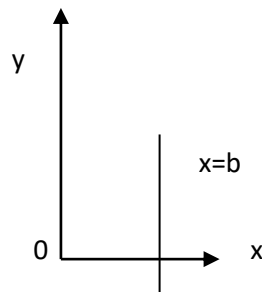
Equation of  $y$  axis i.e.  $oy$  line is  $x=0$

51.



$y=k$  is the equation of a line parallel to  $y$  axis at a distance  $k$  from the origin

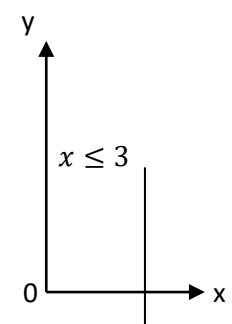
52.



$x=b$  is the equation of a line parallel to  $y$  axis at a distance  $b$  from the origin

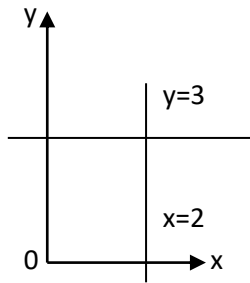
53.  $X=2$  represents a line parallel to  $Y$  axis at a distance 2 units from the origin.

54.



$x \geq 0, x \leq 3$  represents a region between  $Y$  axis and the line  $x=3$ .

55.  $0 \leq x \leq 2, 0 \leq y \leq 3$  represents a rectangular region



56. The linear equation  $ax+by+c=0$  represents a straight line.

57. The line  $x=2$  represents the linear equation  $1 \cdot x+0 \cdot y=2$

58. The line  $Y=4$  represents the linear equation  $0 \cdot X+1 \cdot y=4$

59. The line  $\frac{x}{a} + \frac{y}{b} = 1$  cuts intercepts  $a$  units on  $x$  axis and  $b$  units on  $y$  axis.

60. The line  $3x + 4y=12$  can be put in the form,

$$\frac{x}{4} + \frac{y}{3} = 1$$

The line cuts 4 units intercept on  $x$  axis and 3 units intercept on  $y$  axis.

61. Collection of observations for a preplanned assignment is called data.

62. Marks of 10 students collected as 45, 30, 30, 41, 90, 20, 80, 75, 90, 60 is a data.

63. number of repetitions of a variable  $x$  is called its frequency.

Ex. in the data 45, 30, 45, 41, 10, 30, 45

frequency of 45 is 3

frequency of 30 is 2

frequency of 41 is 1

frequency of 10 is 1

Total frequency for the data = 7

64. Mean of a data  $x_1, x_2, x_3, x_4, x_5$ , is given by mean =  $\frac{x_1+x_2+x_3+x_4+x_5}{\text{Total frequency}(5)}$

65. Mean of the marks 45, 30, 41, 48, 50 is mean =  $\frac{45+30+41+48+50}{5} = \frac{214}{5} = 42.8$

66. Mean of 10, 20, 20, 10, 10

$$= \frac{3 \times 10 + 2 \times 20}{3+2} = \frac{70}{5} = 14$$

67. Median of a data is The value of middle most variable when the data is arranged in ascending or descending order.

68. If a data contains  $n$  observation. Then median is given by  $\frac{n}{2}$ th item when arranged in ascending or descending order.

69. Find median of 45, 30, 41, 48, 50. Arranging the data in ascending order we have 30, 41, 45, 48, 50. – Total 5 observations.

$$\text{Median} = \frac{5}{2} \text{th item} = 2.5 \text{th item} = \text{mean of 2nd and 3rd} = \frac{41+45}{2} = 43$$

70. Mode of a data is the value of the variable having maximum frequency.

71. Mode of the data 45, 30, 31, 45, 20, 50, 45, 60, is 45 ( as 45 has maximum frequency 3)

72. The result of an experiment is called an outcome or event.

73. The experiment is called random if its outcome cannot be predicted in advance.

74. The collection of all possible outcomes of an experiment is called sample space.

75. Two or more events are called equally likely if the chance of their happening is equal.

76. Two events are mutually exclusive if the happening of any one prevents the happening of the other.

77. All possible outcomes of an experiment are called exhaustive events.
78. If an event can occur in  $m$  ways and other event can occur in  $n$  ways then exactly one event can occur in  $m+n$  ways.
79. If a box contains 8 white balls and 4 red balls then either a white or a red ball can be chosen in  $8+4=12$  ways.
80. If there are  $m$  outcomes for an event A and  $n$  outcomes for event B then there are  $mn$  outcomes for event AB.
81. When a pair of dice are rolled. The number of outcomes =  $6 \times 6=36$
82. When two coins are tossed, the number of outcomes favourable to single Head is 2 (i.e. TH, HT)
83. Probability of occurrence of an event A =  $P(A) = \frac{\text{number of favourable events}}{\text{number of possible outcomes}}$
84. When two coins are tossed. Sample space = {HH, HT, TH, TT}  
 Probability of two Heads =  $P(2H)=\frac{1}{4}$ ,  $P(IH) = \frac{2}{4}$
85. Addition theorem.  $P(A \cup B)=P(A)+P(B)-P(A \cap B)$
86. Multiplication Theorem  $P(A \cap B)=P(A) \cdot P(B)$  where A and B are independent events.
87. In a class of 30 students, the probability of selection of one student= $\frac{1}{30}$
88. A dice is thrown, the probability of getting a 5 =  $\frac{1}{6}$
89. Probability of drawing an ace from a pack of cards =  $\frac{4}{52} = \frac{1}{13}$
90. The collection of results of an operation is called population.
91. A part of population is called sample.
92. Probability of drawing an ace of diamond is  $\frac{1}{13}$
93. Probability of obtaining two heads in two throws of a single coin is  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
94.  $(-m) \times (-n) = mn$   
 $(-m) \times n = -(mn)$
95. Two events A and B are said to be independent if occurrence of A will not affect the occurrence of B.
96. A collection containing finite number of objects is called finite population.
97. If 5 is added after dividing 15 by 3 we get 10.
98. If 5 is subtracted after dividing 50 by 5. We get 5.
99. 0 is not a natural number.
100. 0 is an integer, a rational, a real and a complex number.
101.  $-a$  is the negative of  $a$  with respect to addition.

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